The "tipping point" behaviour of epidemics: The N-flu simulator

René Gommes[1]
13 Nov 2005

1 Introduction

The tipping point, how little things can make a big difference by Malcolm Gladwell (2002) is a book[2] about the many socio-economic variables around us that, for no immediately obvious reason, suddenly undergo rapid growth or decline. This includes fashions, book sales (what makes a best-seller?), crime rates in large cities, teenager smoking and many others.

The book has an end-note explaining how density of population plays a critical role in the spreading of epidemics. Below a threshold value, the epidemic dies out, while it expands rapidly at higher densities. Gladwell provides a hypothetical example of a group of tourists visting Manhattan. It is assumed that the tourists are infected with "24 hours flu" and that they use the subway a lot. If they visit during summer, when the subway is not very crowded, nothing happens. If, on the other hand, they visit before christmas when the subway is crowded, there is the potential for starting an epidemic. There is a specific "tipping point" value of the number the tourist come in contact with below which there will be no epidemic. Above the tipping point, an epidemic will be started.

This note decribes a simple model (implemented in a speadsheet) that can be used to simulate the tipping point behaviour.

---

1 rene@gommes.net
2 Black Bay Books (Paperback), 304 pp. The first (hardcover) editions was published in 2000.

2 The algorithm

2.1 General

We assume there are 1000 people living in a given area. People are catching "N-day flu" or "N-flu", a disease that lasts for N days.

The status S of a person is a number between 0 and N, i.e. on the first day of catching the flu, the status is N, and it decreases every day by 1, until the person recovers completely and its N-flu status reaches 0. Since this is just a tipping point toy that has no ambition of being "medically correct", the same person can contract the N-flu again on the same day if she is unlucky!

> **IF $S_{beginning\ of\ day}$ > 0 THEN**
> **$S_{end\ of\ day}$ = $S_{beginning\ of\ day}$ - 1**
> **ELSE**
> **Determine if the person catches N-flu**
> **END**

To determine if the person catches the flu, it is necessary to determine how many people she will meet that day, how many will be infected and how many chances she has to catch the disease if the person met is infected.

With the following notation

> **du = number of persons met per day**
>
> **P = probability that the contact with an infected person will result in an infection [0,1]**
>
> **E$15 = the fraction of infected persons in the total population [0,1]**

a person catches the N-flu if

> **E$15 x P x du >1**

The model further assumes the effect of N-flu does not affect all people to same extent. The full bown N-flu is described by S = N, while some others get affected with S < N and recover in a shorter time.

The IF above thus takes the form

```
IF Sbeginning of day > 0 THEN
        Send of day = Sbeginning of day  - 1
ELSE
        IF 1<E$15*P*du THEN
        Send of day = int ((N+1)*rand)
        ELSE
        Send of day = 0
        ENDIF
ENDIF
```

rand generates a uniformly distributed random number in the interval [0,1] and **int** returns the integer value of the argument, so that **int** ((N+1)***rand**) will be any number between 0 and N.

2.2 Initial conditions

The initial conditions describe the status of the population at the end of day 1 (the simulation runs for 100 days). The user defines the fraction F=[0,1] of people that are affeted by N-flu at the end of the first day.

A specific person reaches status S if **rand** is less or equal to F

```
IF rand<=F THEN
        Send of day = int ((N+1)*rand)
ELSE
        Send of day = 0
ENDIF
```

2.3 Relation between population density and number of contacts between people

This is a critical parameter: it determines how many people any person of the population of 1000 individuals meets per day. Clearly, this can only be determined approximately: what we are interested in in the number of contacts as a function of density.

Two different approaches are followed. In the first, we determine the "minimum distance" **dmin** between two people, and we assume that the

shorter the distance, the larger the number of contacts. Let us
assume the 1000 people live in a square box with a side of 1. We fill
the box with circles that symbolize people, each with a diameter of **R**
(obviously, **R<<1**).
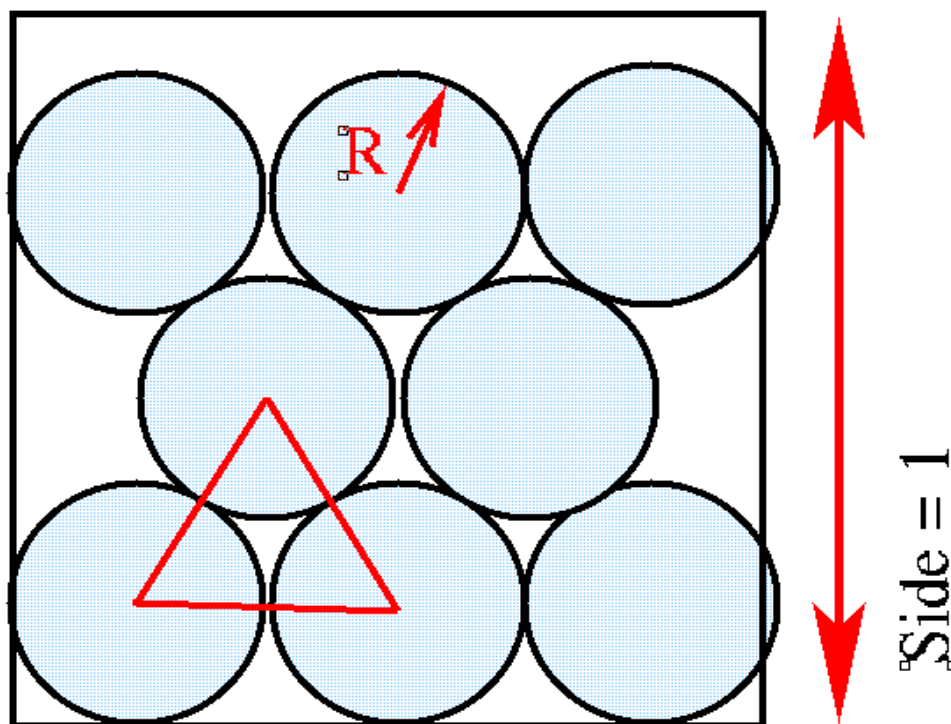
The "area" a person occupies is then πR^2 and $n = 1/ (\pi R^2)$ of them fit
into the box. The sortest distance **dmin** between two people is then
2R. It is proportional to the inverse of the square root of **n**:

$$\text{dmin} \equiv 1 \,/\, \sqrt{n}$$

For the second approach, we consider the same box of side 1 and we
"throw" 2 points (1 and 2) randomly into the box. The distance
between the points is $d_{1\text{-}2}$. If this is done a large number of times,
the average of the distance between two points is approximately
d_{avg}=**0.52,** but the absolute value does not matter here. Now consider
that we throw 3 points into the box. This time, the distances are $d_{1\text{-}2}$, $d_{1\text{-}3}$ and $d_{2\text{-}3}$.   There are thus 3 distances to consider. Since the
average distance between the points is always d_{avg}, the average
distance is d_{avg} divided by the number of distances (3 with 3 points).
All we have to do now is to know how many distances we have to take
into consideration for **n=3, 4, 5** ... points. The number of couples of
poinst that can be created with **n** points is **n(n-1)/2**: 1 with **n=2**, 3

with **n=3**, 6 with **n=4**, 10 with **n=5**, etc. The average distance **dbar** between **n** random points inside a box is thus

$$dbar \equiv 1 \: / \: n \: / \: (n - 1)$$
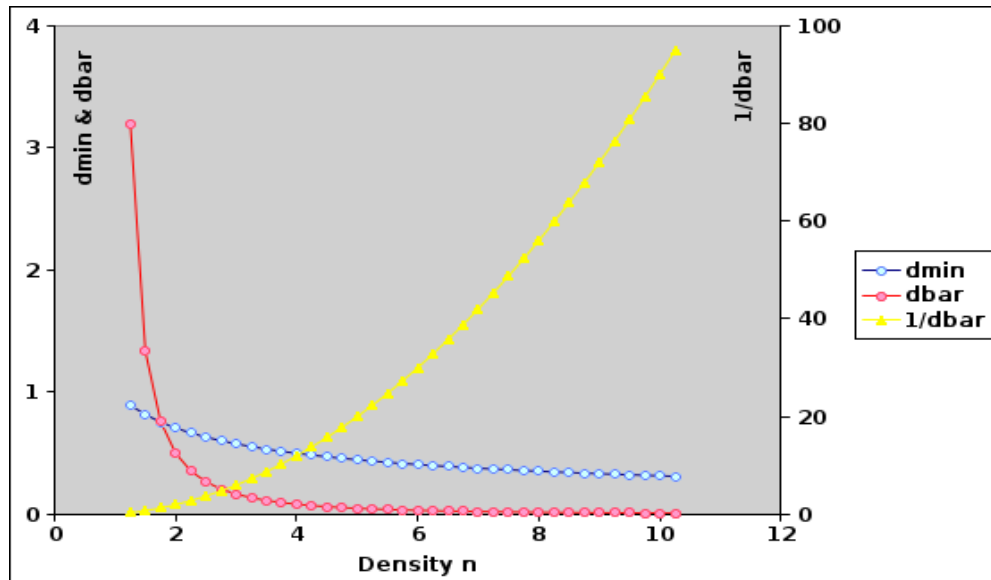
3. Implementation in a spreadsheet

To implement the N-flu simulation, we create a matrix of 1000 lines (the population that will be affected by the N-flu virus) and 100 columns (the 100 days over which we simulate) that will hold the status **S** of every person for every day.

The parameters that control the model are the 4 below:

F   *initial fraction of infected people*
N   *maximum number of days that infection lasts*
P   *probability of getting infected if meeting another infected*
         person
n   *density*

Typical values used for demonstrating the tipping point effect are **F=0.5**, **N=12**, **P=0.2** and **n=11**. Based on the parameters, we further compute **dmin** the distance to closest individual (**dmin=0.3015**), **dbar=0.0091** (average distance between individuals) and the most important factor: d**u=11**, the number of persons met, *e.g.* **0.1/dbar** or **5/dmin**

Note that, in practice, it is useful not to use 1/dbar and 1/dmin without a factor that converts it to a somehow realistic number of people a random individual can meet in a day.

As explained under 2.2, we define the initial status of every individual on day 1. We next create an auxiliary matrix below the status matrix to determine who is infected (1) and who is not (0). The sum of the elements of the auxiliary matrix indicates how many people are affected[3] and the probability that a random person is affected. For instance, if 234 people got N-flu, the probability that a random person from our population of 1000 is 0.234.

We can now proceed to colums 2 (day 2) to 100 and apply the algorith explained under 2.1.

If we have drawn a graph showing how the number of N-flu affected people varies over time, it is now easy to vary the parameters (in particular n, the population density) and observe that, there is a critical value below which the epidemic dies out, while it grows rapidly above the threshold.

The implementation of the N-flu model with Gnumeric could look like the figure below. The values that can be changed by the user are in the blue area

---

3 Or "infected!"
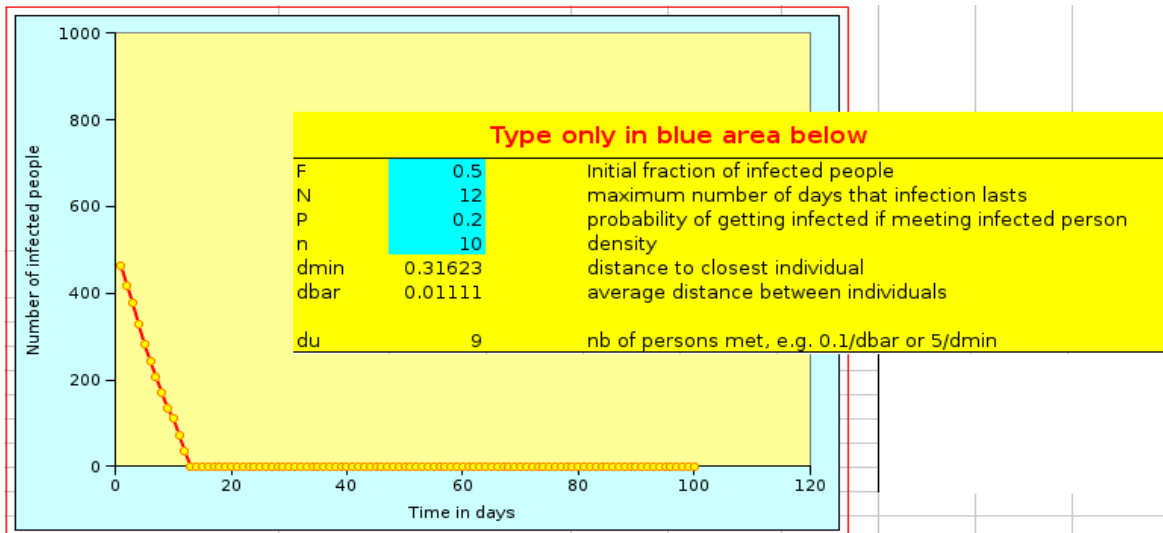
				initial population					
	prob(person is infected) at beg of day				0.462	0.418	0.376	0.328	
	number infected at end of day			462	418	376	328	282	
	days			1	2	3	4	5	
	people	1	0	0	0	0	0		
		2	3	2	1	0	0		
		3	6	5	4	3	2		
		4	4	3	2	1	0		
		5	4	3	2	1	0		
		6	3	2	1	0	0		

The figure on the next page shows how varying only the density from
11 to 12 completely changes the situation from the epidemic dying out
to a sustained epidemic.